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CHAPTER

4 Parallel Forces and Couples

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4.1. INTRODUCTION

In the previous chapters, we have been studying forces acting at one point. But, sometimes, the given forces have their lines of action parallel to each other. A little consideration will show, that such forces do not meet at any point, though they do have some effect on the body on which they act. The forces, whose lines of action are parallel to each other, are known as parallel forces.

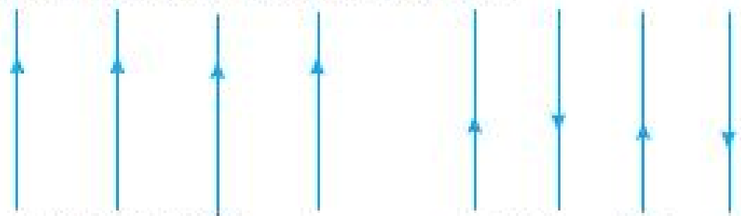
4.2. CLASSIFICATION OF PARALLEL FORCES

The parallel forces may be, broadly, classified into the following two categories, depending upon their directions :

1. Like parallel forces.
2. Unlike parallel forces.

4.3. LIKE PARALLEL FORCES

The forces, whose lines of action are parallel to each other and all of them act in the same direction as shown in Fig. 4.1 (*a*) are known as like parallel forces.



(*a*) Like parallel forces

(*b*) Unlike parallel forces.

Fig. 4.1.

4.4. UNLIKE PARALLEL FORCES

The forces, whose lines of action are parallel to each other and all of them do not act in the same direction as shown in Fig. 4.1 (*b*) are known as unlike parallel forces.

4.5. METHODS FOR MAGNITUDE AND POSITION OF THE RESULTANT OF PARALLEL FORCES

The magnitude and position of the resultant force, of a given system of parallel forces (like or unlike) may be found out analytically or graphically. Here we shall discuss both the methods one by one.

4.6. ANALYTICAL METHOD FOR THE RESULTANT OF PARALLEL FORCES

In this method, the sum of clockwise moments is equated with the sum of anticlockwise moments about a point.

Example 4.1. *Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.*

Solution. Given : The system of given forces is shown in Fig. 4.2

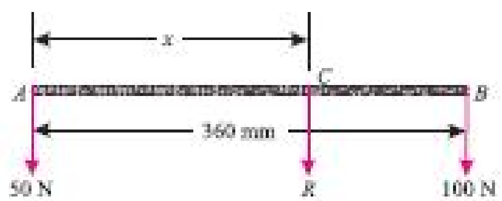


Fig. 4.2.

Magnitude of the resultant force

Since the given forces are like and parallel, therefore magnitude of the resultant force,

$$R = 50 + 100 = 150 \text{ N} \quad \textbf{Ans.}$$

Point where the resultant force acts

Let x = Distance between the line of action of the resultant force (R) and A (*i.e.* AC) in mm.

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Now taking clockwise and anticlockwise moments of the forces about C and equating the same,

$$50 \times x = 100\,(360 - x) = 36\,000 - 100\,x$$

or $$150\,x = 36\,000$$

$$\therefore \quad x = \frac{36\,000}{150} = 240 \text{ mm} \quad \textbf{Ans.}$$

Example 4.2. *A beam 3 m long weighing 400 N is suspended in a horizontal position by two vertical strings, each of which can withstand a maximum tension of 350 N only. How far a body of 200 N weight be placed on the beam, so that one of the strings may just break ?*

Solution. The system of given forces is shown in Fig. 4.3.

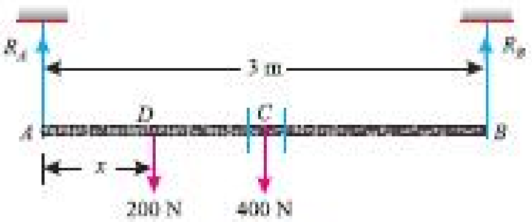


Fig. 4.3.

Let x = Distance between the body of weight 200 N and support A.

We know that one of the string (say A) will just break, when the tension will be 350 N. (*i.e.*, *R_A = 350 N). Now taking clockwise and anticlockwise moments about B and equating the same,

$$350 \times 3 = 200\,(3 - x) + 400 \times 1.5$$

or $$1\,050 = 600 - 200\,x + 600 = 1200 - 200\,x$$

$$\therefore \quad 200\,x = 1\,200 - 1\,050 = 150$$

or $$x = \frac{150}{200} = 0.75 \text{ m} \quad \textbf{Ans.}$$

Example 4.3. *Two unlike parallel forces of magnitude 400 N and 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.*

Solution. Given : The system of given force is shown in Fig. 4.4

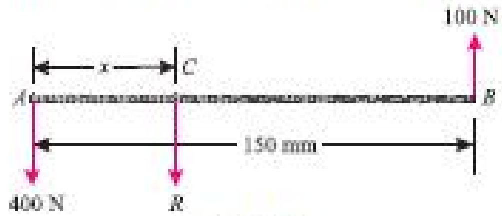


Fig. 4.4.

Magnitude of the resultant force

Since the given forces are unlike and parallel, therefore magnitude of the resultant force,

$$R = 400 - 100 = 300 \text{ N} \quad \textbf{Ans.}$$

* The procedure for finding the reaction at either end will be discussed in the chapter on 'Support Reactions'.

Point where the resultant force acts

Let x = Distance between the lines of action of the resultant force and A in mm.

Now taking clockwise and anticlockwise moments about A and equating the same,

$$300 \times x = 100 \times 150 = 15\,000$$

$$\therefore \qquad x = \frac{15\,000}{300} = 50 \text{ mm} \qquad \textbf{Ans.}$$

Example 4.4. *A uniform beam AB of weight 100 N and 6 m long had two bodies of weights 60 N and 80 N suspended from its two ends as shown in Fig. 4.5.*

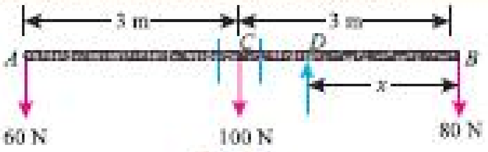


Fig. 4.5.

Find analytically at what point the beam should be supported, so that it may rest horizontally.

Solution. Given : Weight of rod AB = 100 N ; Length of rod AB = 6 mm and weight of the bodies supported at A and B = 60 N and 80 N.

Let x = Distance between B and the point where the beam should be supported.

We know that for the beam to rest horizontally, the moments of the weights should be equal. Now taking moments of the weights about D and equating the same,

$$80x = 60\,(6 - x) + 100\,(3 - x)$$

$$= 360 - 60x + 300 - 100x = 660 - 160x$$

$$240x = 660$$

or $$x = \frac{660}{240} = 2.75 \text{ m} \qquad \textbf{Ans.}$$

4.7. GRAPHICAL METHOD FOR THE RESULTANT OF PARALLEL FORCES

Consider a number of parallel forces (say three like parallel forces) P_1, P_2 and P_3 whose resultant is required to be found out as shown in Fig. 4.6 (a).

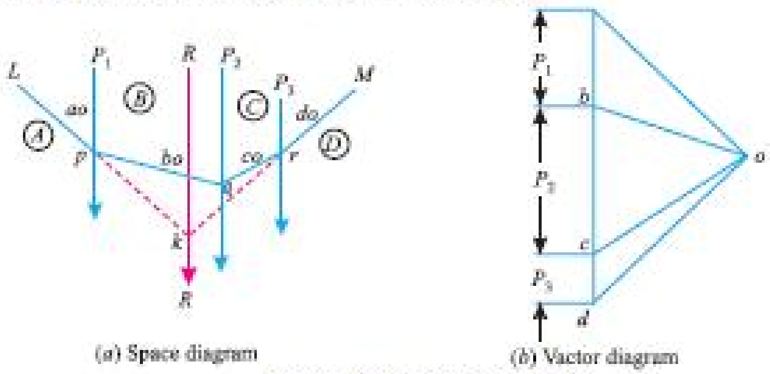


Fig. 4.6. Resultant of parallel forces

First of all, draw the space diagram of the given system of forces and name them according to Bow's notations as shown in Fig. 4.6 (a). Now draw the vector diagram for the given forces as shown in Fig. 4.6 (b) and as discussed below :

1. Select some suitable point a, and draw ab equal to the force AB (P_1) and parallel to it to some suitable scale.

2. Similarly draw *bc* and *cd* equal to and parallel to the forces *BC* (P_2) and *CD* (P_3) respectively.
3. Now take some convenient point *o* and joint *oa*, *ob*, *oc* and *od*.
4. Select some point *p*, on the line of action of the force *AB* of the space diagram and through it draw a line *Lp* parallel to *ao*. Now through *p* draw *pq* parallel to *bo* meeting the line of action of the force *BC* at *q*.
5. Similarly draw *qr* and *rM* parallel to *co* and *do* respectively.
6. Now extend *Lp* and *Mr* to meet at *k*. Through *k*, draw a line parallel to *ad*, which gives the required position of the resultant force.
7. The magnitude of the resultant force is given by *ad* to the scale.

Note. This method for the position of the resultant force may also be used for any system of forces *i.e.* parallel, like, unlike or even inclined.

Example 4.5. *Find graphically the resultant of the forces shown in Fig. 4.7. The distances between the forces are in mm.*

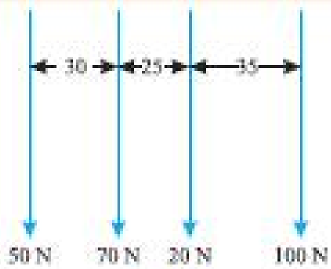
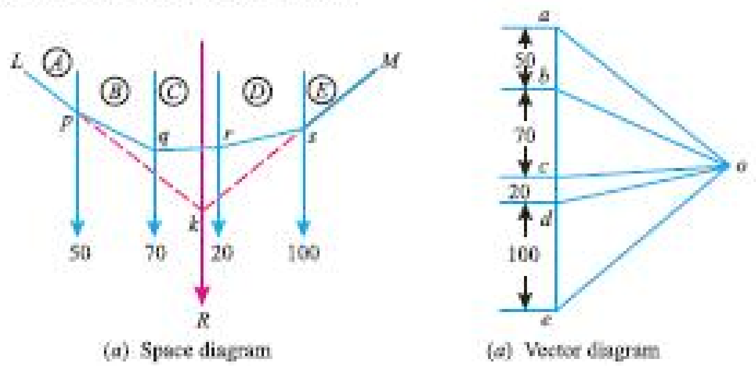


Fig. 4.7.

Also find the point, where the resultant acts.

Solution. Given : forces : 50 N, 70 N, 20 N and 100 N.

First of all, draw the space diagram for the given system of forces and name them according to Bow's notations as shown in Fig. 4.8 (*a*)



(*a*) Space diagram (*a*) Vector diagram

Fig. 4.8.

Now draw the vector diagram for the given forces as shown in Fig. 4.8 (*b*) and as discussed below :

1. Take some suitable point *a* and draw *ab* equal and parallel to force *AB* (*i.e.* 50 N) to some scale. Similarly draw *bc* equal to the force *BC* (*i.e.* 70 N), *cd* equal to the force *CD* (*i.e.* 20 N) and *de* equal to the force *DE* (*i.e.* 100 N) respectively.

2. Now select some suitable point *o*, and join *oa*, *ob*, *oc*, *od* and *oe*.
3. Now take some suitable point *p* on the line of action of the force *AB* of the space diagram. Through *p* draw a line *Lp*, parallel to *ao* of the vector diagram.
4. Now, through *p*, draw *pq* parallel to *bo*, meeting, the line of action of the force *BC* at *q*. Similarly, through *q* draw *qr* parallel to *co*, through *r* draw *rs* parallel to *do* and through *s* draw *sM* parallel to *eo*.
5. Now extend the lines *Lp* and *Ms* meeting each other at *k*. Through *k* draw a line parallel to *ae* which gives the required position of the resultant force.
6. By measurement, we find that resultant force,

$$R = ae = 240 \text{ N} \quad \textbf{Ans.}$$

and line of action of *k* from force *AB* = 51 mm **Ans.**

Example 4.6. *Find graphically the resultant of the forces shown in Fig. 4.9*

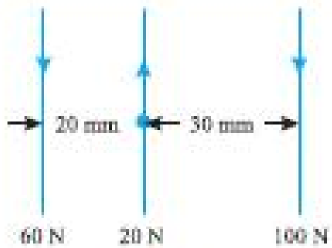
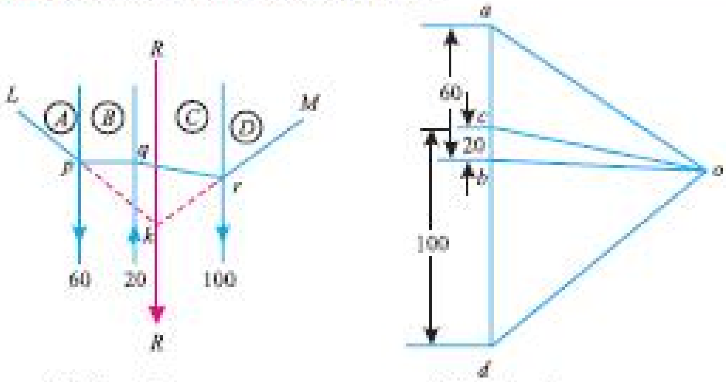


Fig. 4.9.

Also find the point where the resultant force acts.

Solution. Given forces : 60 N; 20 N; and 100 N.



(*a*) Space diagram (*b*) Vector diagram

Fig. 4.10.

First of all, draw the space diagram for the given system of forces and name them according to Bow's notations as shown in Fig. 4.10 (*a*).

It may be noted that the force *AB* (equal to 60 N) is acting downwards, force *BC* (equal to 20 N) is acting upwards and the force *CD* (equal to 100 N) is acting downwards as shown in the figure. Now draw the vector diagram for the given forces as shown in Fig. 4.10 (*b*) and as discussed below :

1. Take some suitable point *a* and draw *ab* equal and parallel to force *AB* (*i.e.*, 60 N) to some scale. Similarly, draw *bc* (upwards) equal to force *BC* (*i.e.* 20 N) and *cd* equal to the force *CD* (*i.e.*, 100 N) respectively.

2. Now select some suitable point *o* and join *oa*, *ob*, *oc* and *od*.
3. Now take some suitable point *p* on the line of action of the force *AB* of the space diagram. Through *p* draw a line *Lp* parallel to *ao* of the vector diagram.
4. Now through *p*, draw *pq* parallel to *bo* meeting the line of action of the force *BC* at *q*. Similarly through *q* draw *qr* parallel to *co*. Through *r* draw *rM* Parallel to *do*.
5. Now extend the lines *Lp* and *Mr* meeting each other at *k*. Through *k* draw a line parallel to *ad*, which gives the required resultant force.
6. By measurement, we find that resultant force,

$$R = ad = 140 \text{ N} \qquad \textbf{Ans.}$$

and line of action of *k* from force *AB* = 33 mm **Ans.**

Note. In some cases, the lines *Lp* and *rM* are parallel and do not meet each other. This happens, when magnitude of the sum of upward forces is equal to sum of the downward forces.

EXERCISE 4.1

1. Two like parallel forces of 10 N and 30 N act at the ends of a rod 200 mm long. Find magnitude of the resultant force and the point where it acts. [**Ans.** 40 N ; 150 mm]
2. Find the magnitude of two like parallel forces acting at a distance of 240 mm, whose resultant is 200 N and its line of action is at a distance of 60 mm from one of the forces. [**Ans.** 50 N ; 150 N]

   Hint.
   $$P + Q = 200$$
   $$Q \times 240 = 200 \times 60 = 12\,000$$
   $$\therefore \quad Q = 50 \text{ N and } P = 200 - 50 = 150 \text{ N}$$
3. Two unlike parallel forces are acting at a distance of 450 mm from each other. The forces are equivalent to a single force of 90 N, which acts at a distance of 200 mm from the greater of the two forces. Find the magnitude of the forces. [**Ans.** 40 N ; 130 N]
4. Find graphically the resultant force of the following like parallel forces :
   P_1 = 20 N ; P_2 = 50 N ; P_3 = 60 N and P_4 = 70 N
   Take distances between P_1 and P_2 as 40 mm, between P_2 and P_3 as 30 mm and between P_3 and P_4 as 20 mm.
   [**Ans.** 200 N ; 62.5 mm]

4.8. COUPLE

A pair of two equal and unlike parallel forces (*i.e.* forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

A couple is a pair of forces applied to the key of a lock.

4.9. ARM OF A COUPLE

The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig. 4.11.

Fig. 4.11.

4.10. MOMENT OF A COUPLE

The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple. Mathematically:

Moment of a couple $= P \times a$

where P = Magnitude of the force, and

a = Arm of the couple.

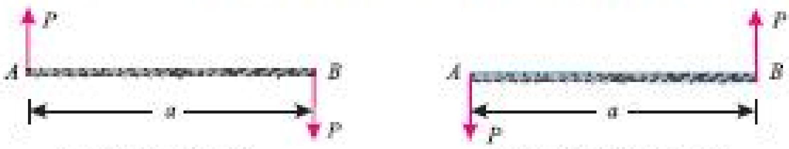
4.11. CLASSIFICATION OF COUPLES

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts :

1. Clockwise couple, and 2. Anticlockwise couple.

4.12. CLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 4.12 (a). Such a couple is also called positive couple.



(a) Clockwise couple (b) Anticlockwise couple

Fig. 4.12.

4.13. ANTICLOCKWISE COUPLE

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 4.12 (b). Such a couple is also called a negative couple.

4.14. CHARACTERISTICS OF A COUPLE

A couple (whether clockwise or anticlockwise) has the following characteristics :

1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of coplaner couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Example 4.7. *A square ABCD has forces acting along its sides as shown in Fig. 4.13. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.*

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions must be zero. Resolving the forces horizontally,

$$100 - 100 \cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100 \cos 45° \text{ N}$$

$$= 100 - (100 \times 0.707) = 29.3 \text{ N} \quad \textbf{Ans.}$$

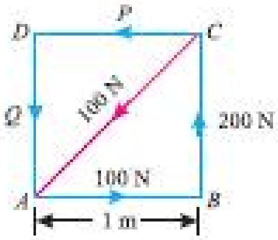


Fig. 4.13.

Now resolving the forces vertically,

$$200 - 100 \sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - (100 \times 0.707) = 129.3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the couple

We know that moment of the couple is equal to the algebraic sum of the moments about any point. Therefore moment of the couple (taking moments about A)

$$= (-200 \times 1) + (-P \times 1) = -200 - (29.3 \times 1) \text{ N-m}$$

$$= -229.3 \text{ N-m} \quad \textbf{Ans.}$$

Since the value of moment is negative, therefore the couple is anticlockwise.

Example 4.8. *ABCD is a rectangle, such that AB = CD = a and BC = DA = b. Forces equal to P act along AD and CB and forces equal to Q act along AB and CD respectively. Prove that the perpendicular distance between the resultants of P and Q at A and that of P and Q at C*

$$= \frac{(P \times a) - (Q \times b)}{\sqrt{(P^2 + Q^2)}}$$

Solution. Given : The system of forces is shown in Fig. 4.14.

Let $\quad x$ = Perpendicular distance between the two resultants.

We know that the resultant of the forces P and Q at A,

$$R_1 = \sqrt{P^2 + Q^2} \quad ...(i)$$

and resultant of the forces P and Q at C,

$$R_2 = \sqrt{P^2 + Q^2} \quad ...(ii)$$

$\therefore$ Resultant $R = R_1 = R_2$ ...[from equations (*i*) and (*ii*)]

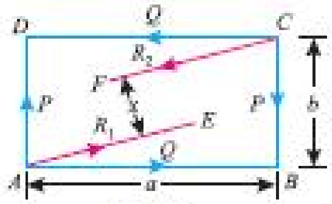


Fig. 4.14.

We know that moment of the force (P) about A,

$$M_1 = P \times a \quad ...(+\text{ Due to clockwise})$$

and moment of the force (Q) about A,

$$M_2 = -Q \times b \quad ...(-\text{ Due to anticlockwise})$$

$\therefore$ Net moment of the two couples

$$= (P \times a) - (Q \times b) \quad ...(iii)$$

and moment of the couple formed by the resultants

$$= R \times x = \sqrt{P^2 + Q^2} \times x \quad ...(iv)$$