SEMESTER:II

BRANCH:MECHANICAL ENGINEERING

SUBJECT:ENGINEERING MECHANICS

TOPIC: CHAPTER 9:WORK,POWER AND ENERGY

Work
When a force acts on an object and the object actually moves in the direction of force, then the work is said to be done by the force.

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

If under a constant force F the object displaced through a distance s, then work done by the force

$W = F * s = F s \cos \theta$

where a is the smaller angle between F and s.

Work is a scalar quantity, Its S1 unit is joule and CGS unit is erg.

$\therefore$ 1 joule = 10^7 erg
Its dimensional formula is $[ML^2T^{-2}]$.
Work done by a force is zero, if

(a) body is not displaced actually, i.e., $s = 0$

(b) body is displaced perpendicular to the direction of force, i.e.,

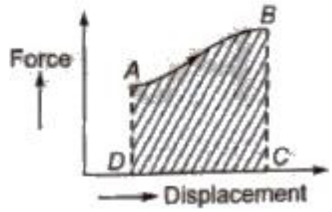
$\theta = 90°$

Work done by a force is **positive** if angle between F and s is acute angle.
Work done by a force is **negative** if angle between F and s is obtuse angle.
Work done by a constant force depends only on the initial and final Positions and not on the actual path followed between initial and final positions.

Work done in different conditions
(i) Work done by a variable force is given by

$W = \int F * ds$

It is equal to the area under the force-displacement graph along with proper sign.



Work done = Area ABCDA

(ii) Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.

(iii) In equilibrium (static or dynamic), the resultant force is zero therefore resultant work done is zero.

(iv) If work done by a force during a rough trip of a system is zero, then the force is conservative, otherwise it is called non-conservative force.

- Gravitational force, electrostatic force, magnetic force, etc are conservative forces. All the central forces are conservative forces.
- Frictional force, viscous force, etc are non-conservative forces.

(v) Work done by the force of gravity on a particle of mass m is given by W = mgh

where g is acceleration due to gravity and h is height through particle one displaced.

(vi) Work done in compressing or stretching a spring is given by

$W = 1 / 2\ kx^2$
where k is spring constant and x is displacement from mean position.

(vii) When on end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal

table from $x = x_1$ to $x = x_2$ then $W = 1 / 2\ k\ (x^2x_2 – x^2x_1)$
(viii) Work done by the couple for an angular displacement θ is given by W = i * θ

where i is the torque of the couple.

power
The time rate of work done by a body is called its power.

Power = Rate of doing work = Work done / Time taken

If under a constant force F a body is displaced through a distance **s** in time t, the power
p = W / t = F * s / t

But s / t = v ; uniform velocity with which body is displaced.

∴ P = F * v = F v cos θ

where θ is the smaller angle between F and v.

power is a scalar quantity. Its S1 unit is watt and its dimensional formula is $[ML^2T^{-3}]$.
Its other units are kilowatt and horse power,

1 kilowatt = 1000 watt

1 horse power = 746 watt

Energy
Energy of a body is its capacity of doing work.

It is a scalar quantity.

Its S1 unit is joule and CGS unit is erg. Its dimensional formula is $[ML^3T^{-3}]$.
There are several types of energies, such as mechanical energy (kinetic energy and potential energy), chemical energy, light energy, heat energy, sound energy, nuclear energy, electric energy etc.

Mechanical Energy
The sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as law of conservation of mechanical energy.

Mechanical energy is of two types:

1. **Kinetic Energy**
The energy possessed by any object by virtue of its motion is called its kinetic energy.

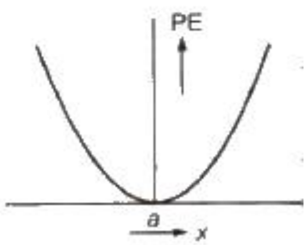
Kinetic energy of an object is given by

$k = 1 / 2\ mv^2 = p^2 / 2m$
where m = mass of the object, U = velocity of the object and p = mv = momentum of the object.

2. **Potential Energy**
The energy possessed by any object by virtue of its position or configuration is called its potential energy.

There are three important types of potential energies:

(i) **Gravitational Potential Energy** If a body of mass m is raised through a height h against gravity, then its gravitational potential energy = mgh,



(ii) **Elastic Potential Energy** If a spring of spring constant k is stretched through a distance x. then elastic potential energy of the spring = $1 . 2\ kx^2$
The variation of potential energy with distance is shown in figure.

Potential energy is defined only for conservative forces. It does not exist for non-conservative forces.

Potential energy depends upon frame of reference.

(iii) **Electric Potential Energy** The electric potential energy of two point charges ql and q'l. separated by a distance r in vacuum is given by
$U = 1 / 4\pi\Sigma_0 * q_1 q_2 / r$
Here $1 / 4\pi\Sigma_0 = 9.0 * 10^{10}$ N-m^2 / C^2 constant.

Work-Energy Theorem
Work done by a force in displacing a body is equal to change in its kinetic energy.

$$W = \int_{v_1}^{v_2} F \cdot ds = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = K_f - K_i = \Delta\text{KE}$$

where, K_i = initial kinetic energy
and K_f = final kinetic energy.
Regarding the work-energy theorem it is worth noting that

(i) If W_{net} is positive, then $K_f – K_i$ = positive, i.e., $K_f > K_i$ or kinetic energy will increase and vice-versa.
(ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as:

Work done by all the forces (including the Pseudo force) = change in kinetic energy in non-inertial frame.

Mass-Energy Equivalence
According to Einstein, the mass can be transformed into energy and vice – versa.

When Δm. mass disappears, then produced energy

$E = \Delta mc^2$
where c is the speed of light in vacuum.

Principle of Conservation of Energy
The sum of all kinds of energies in an isolated system remains constant at all times.

Principle of Conservation of Mechanical Energy
For conservative forces the sum of kinetic and potential energies of any object remains constant throughout the motion.

According to the quantum physics, mass and energy are not conserved separately but are conserved as a single entity called ‘mass-energy’.

QUESTION:

Example of work

An object is horizontally dragged across the surface by a 100 N force acting parallel to the surface. Find out the amount of work done by the force in moving the object through a distance of 8 m

We know that,

F = 100 N

d = 8 m

Since F and d are in the same direction,

θ = 0, [θ is the angle of the force to the direction of movement]

W = F Cos θ

= 100 x 8 x Cos 0

= 800 J [Since Cos 0 = 1]

QUESTION:

A garage hoist lifts a truck up 2 meters above the ground in 15 seconds. Find the power delivered to the truck. [Given: 1000 kg as the mass of the truck]

First we need to calculate the work done, which requires the force necessary to lift the truck against gravity:

F = mg = 1000 x 9.81 = 9810 N.

W = Fd = 9810N x 2m = 19620 Nm = 19620 J.

The power is P = W/t = 19620J / 15s = 1308 J/s = 1308 W. P=f.v

1. A force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$ acts on a particle and displaces it through a distance $\vec{S} = 4\hat{i} + 6\hat{j}$ Calculate the work done if force and work done are in the same direction.

Solution:

Force $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$

Distance $\vec{S} = 4\hat{i} + 6\hat{j}$

Work done = $\vec{F} \cdot \vec{S} = (\hat{i} + 2\hat{j} + 3\hat{k}).(4\hat{i} + 6\hat{j})$

$= 4+12+0 = 16$ J

2. A particle moves along X- axis from x=0 to x=8 under the influence of a force given by F= $3x^2 - 4x + 5$. Find the work done in the process.

Solution:

Work done in moving a particle from x=0 to x=8 will be

$$W = \int_0^8 F dx = \int_0^8 (3x^2 - 4x + 5) dx = \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x\right]_0^8$$

$$W = \left[3\frac{(8)^3}{3} - 4\left(\frac{8^2}{2}\right) + 40\right]$$

$$= [512 - 128 + 40] = 424 J$$

2. A particle moves along X- axis from x=0 to x=8 under the influence of a force given by
5. Find the work done in the process.

Solution:

Work done in moving a particle from x=0 to x=8 will be

$$W = \int_0^8 F dx = \int_0^8 (3x^2 - 4x + 5) dx = \left[\frac{3x^3}{3} - \frac{4x^2}{2} + 5x \right]_0^8$$

$$W = \left[3\frac{(8)^3}{3} - 4\left(\frac{8^2}{2}\right) + 40 \right]$$
$$= [512 - 128 + 40] = 424 J$$

3. A body of mass 10kg at rest is subjected to a force of 16N. Find the kinetic energy at the end of 10 s.

Solution:

Mass m = 10 kg

Force F = 16 N

time t = 10 s

$$a = F/m = \frac{16N}{10kg} = 1.6 ms^{-2}$$

we know that, $v = u + at$

$$= 0 + 1.6 \times 10 = 16\ m\ s^{-1}$$

Kinetic energy K.E $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 10 \times 16 \times 16$$

$$= 1280 J$$

4. A body of mass 5kg is thrown up vertically with a kinetic energy of 1000 J. If acceleration due to gravity is 10ms^{-2}, find the height at which the kinetic energy becomes half of the original value.

Solution:

Mass m = 5kg

K.E E = 1000J

g = 10$m\,s^{-2}$

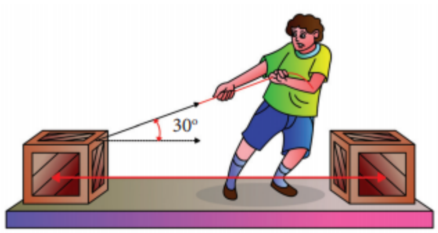
At a height 'h', mgh = $\frac{E}{2}$

$$5 \times 10 \times h = \frac{1000}{2}$$

$$h = \frac{500}{50} = 10m$$

Example 4.1

A box is pulled with a force of 25 N to produce a displacement of 15 m. If the angle between the force and displacement is 30°, find the work done by the force.



Solution

Force, F = 25 N

Displacement, dr = 15 m

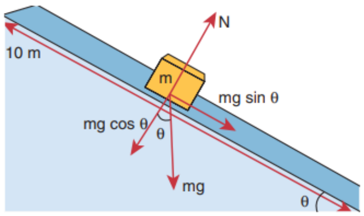
Angle between F and dr, θ = 30°

Work done, W = Fdr cos**θ**

$$W = 25 \times 15 \times \cos 30 = 25 \times 15 \times \frac{\sqrt{3}}{2}$$

Example 4.3

An object of mass *m*=1 kg is sliding from top to bottom in the frictionless inclined plane of inclination angle $\theta = 30^o$ and the length of inclined plane is 10 m as shown in the figure. Calculate the work done by gravitational force and normal force on the object. Assume acceleration due to gravity, $g = 10\ m\ s^{-2}$



Solution

We calculated in the previous chapter that the acceleration experienced by the object in the inclined plane as g sinθ .

According to Newton's second law, the force acting on the mass along the inclined plane F = mg sinθ. Note that this force is constant throughout the motion of the mass.

The work done by the parallel component of gravitational force (mg sinθ) is given by

$$W = \vec{F}.d\vec{r} = Fdr\cos\phi$$

where ϕ is the angle between the force (mg sin **θ)** and the direction of motion (dr). In this case, force (mg sin **θ)** and the displacement ($d\vec{r}$) are in the same direction. Hence $\phi = o$ and $\cos\phi = 1$

$$W = F\ dr = (mg\ \sin\theta)\ (dr)$$

(dr = length of the inclined place)

$$W = 1 \times 10 \times \sin(30^\circ) \times 10 = 100 \times \frac{1}{2} = 50\ J$$

Example 4.4

If an object of mass 2 kg is thrown up from the ground reaches a height of 5 m and falls back to the Earth (neglect the air resistance). Calculate

a) The work done by gravity when the object reaches 5 m height

b) The work done by gravity when the object comes back to Earth

c) Total work done by gravity both in upward and downward motion and mention the physical significance

Example 4.5

A weight lifter lifts a mass of 250 kg with a force 5000 N to the height of 5 m.

a. What is the workdone by the weight lifter?

b. What is the workdone by the gravity?

c. What is the net workdone on the object?

Solution

a. When the weight lifter lifts the mass, force and displacement are in the same direction, which means that the angle between them $\theta = 0^0$. Therefore, the work done by the weight lifter,

$$W_{\text{weight lifter}} = F_w h \cos\theta = F_w h(\cos 0^0)$$
$$= 5000 \times 5 \times (1) = 25{,}000 \text{ joule} = 25 \text{ kJ}$$

b. When the weight lifter lifts the mass, the gravity acts downwards which means that the force and displacement are in opposite direction. Therefore, the angle between them $\theta = 180^0$

$$W_{\text{gravity}} = F_g h \cos\theta = mgh(\cos 180^0)$$
$$= 250 \times 10 \times 5 \times (-1)$$

b. When the weight lifter lifts the mass, the gravity acts downwards which means that the force and displacement are in opposite direction. Therefore, the angle between them $\theta = 180^0$

$$W_{\text{gravity}} = F_g h \cos\theta = mgh(\cos 180^0)$$
$$= 250 \times 10 \times 5 \times (-1)$$
$$= -12{,}500 \text{ joule} = -12.5 \text{ kJ}$$

c. The net workdone (or total work done) on the object

$$W_{\text{net}} = W_{\text{weight lifter}} + W_{\text{gravity}}$$
$$= 25 \text{ kJ} - 12.5 \text{ kJ} = +12.5 \text{ kJ}$$

Example 4.7

Two objects of masses 2 kg and 4 kg are moving with the same momentum of 20 kg m s^{-1}.

a. Will they have same kinetic energy?

b. Will they have same speed?

Solution

a. The kinetic energy of the mass is given by

by $KE = \frac{p^2}{2m}$

For the object of mass 2 kg, kinetic energy is $KE_1 = \frac{(20)^2}{2 \times 2} = \frac{400}{4} = 100\,J$

For the object of mass 4 kg, kinetic energy is $KE_2 = \frac{(20)^2}{2 \times 4} = \frac{400}{8} = 50\,J$

Note that $KE_1 \neq KE_2$ i.e., even though both are having the same momentum, the kinetic energy of both masses is not the same. The kinetic energy of the heavier object has lesser kinetic energy than smaller mass. It is because the kinetic energy is inversely proportional to the mass ($KE \propto 1/m$) for a given momentum.

b. As the momentum, $p = mv$, the two objects will not have same speed.

Solved Example Problems for Potential Energy

Example 4.8

An object of mass 2 kg is taken to a height 5 m from the ground $g = 10ms^{-2}$.

a. Calculate the potential energy stored in the object.

b. Where does this potential energy come from?

c. What external force must act to bring the mass to that height?

d. What is the net force that acts on the object while the object is taken to the height 'h'?

Solution

a. The potential energy $U = m\,g\,h = 2 \times 10 \times 5 = 100$ J

Here the positive sign implies that the energy is stored on the mass.

b. This potential energy is transferred from external agency which applies the force on the mass.

c.

The external applied force $\vec{F}_a$ which takes the object to the height 5 m is $\vec{F}_a = -\vec{F}_g$

$$\vec{F}_a = -(-mg\hat{j}) = mg\hat{j}$$

d. From the definition of potential energy, the object must be moved at constant velocity. So the net force acting on the object is zero.

$$\vec{F}_g + \vec{F}_a = 0$$

Example 4.12

Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction ? Comment on the result. (Assume g = 10 ms^{-2})

Solution

m = 2 kg, d = 10 m, F_{ext} = 20 N, μ_k = 0.9. When an object is in motion on the horizontal surface, it experiences two forces.

a. External force, F_{ext} = 20 N

b. Kinetic friction,

$f_k = \mu_k mg$ = 0.9x(2)x10=18N.

The work done by the external force W_{ext} = Fs = 20x20 =200J

The work done by the force of kinetic friction $W_k = f_k d$ = (-18) x10=-180J Here the negative sign implies that the force of kinetic friction is pposite to the direction of displacement.

The total work done on the object

W_{total} = Wext + W_k = 200 J – 180 J = 20 J .

Since the friction is a non-conservative force, out of 200 J given by the external force, the 180 J is lost and it can not be recovered.

Solved Example Problems for Law of conservation of energy

Example 4.13

An object of mass 1 kg is falling from the height h = 10 m. Calculate

a. The total energy of an object at h =10 m

b. Potential energy of the object when it is at h = 4 m

c. Kinetic energy of the object when it is at h = 4 m

d. What will be the speed of the object when it hits the ground?

(Assume g =10 m s^{-2})

Solution

a. The gravitational force is a conservative force. So the total energy remains constant throughout the motion. At h =10 m, the total energy$E$ is entirely potential energy.

$$E = U = mgh = 1 \times 10 \times 10 = 100 \text{ J}$$

b. The potential energy of the object at h = 4 m is

b. The potential energy of the object at $h = 4$ m is

$$U = mgh = 1 \times 10 \times 4 = 40\,\text{J}$$

c. Since the total energy is constant throughout the motion, the kinetic energy at h = 4 m must be $KE = E - U = 100 - 40 = 60$J

Alternatively, the kinetic energy could also be found from velocity of the object at 4 m. At the height 4 m, the object has fallen through a height of 6 m.

The velocity after falling 6 m is calculated from the equation of motion,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 6} = \sqrt{120}\ \text{m s}^{-1};$$
$$v^2 = 120$$

The kinetic energy is $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 120 = 60\,\text{J}$

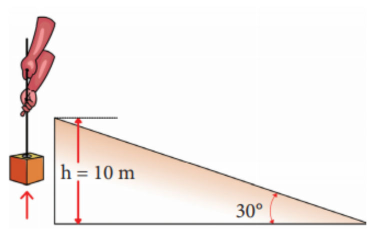
d. When the object is just about to hit the ground, the total energy is completely kinetic and the potential energy, $U = 0$.

$$E = KE = \frac{1}{2}mv^2 = 100\ \text{J}$$

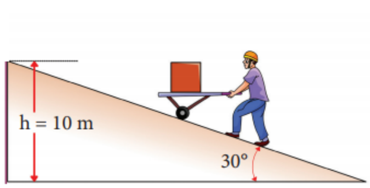
$$v = \sqrt{\frac{2}{m}KE} = \sqrt{\frac{2}{1} \times 100} = \sqrt{200} \approx 14.12\ \text{m s}^{-1}$$

Example 4.14

A body of mass 100 kg is lifted to a height 10 m from the ground in two different ways as shown in the figure. What is the work done by the gravity in both the cases? Why is it easier to take the object through a ramp?



Path (1) straight up



Path (2) along the ramp

Solution

m = 100 kg, h = 10 m

Along path (1):

The minimum force F_1 required to move the object to the height of 10 m should be equal to the gravitational force, F_1 mg = 100 x 10 = 1000 N

The distance moved along path (1) is, =10 m

The work done on the object along path (1) is

W = Fh = 1000 x 10 = 10,000 J

Along path (2):

In the case of the ramp, the minimum force F_2 that we apply on the object to take it up is not equal to mg, it is rather equal to $mg \sin\theta$.(mg sin < mg) .

Here, angle $\theta = 30^o$

Therefore, $F_2 = mg \sin\theta = 100 \times 10 \times \sin 30^o = 100 \times 10 \times 0.5 = 500N$

Hence, ($mg \sin\theta < mg$)

The path covered along the ramp is,

l = h/sin30 = 10/0.5 =20m

The work done on the object along path (2) is, W = F2 l = 500 × 20 = 10,000 J

Since the gravitational force is a conservative force, the work done by gravity on the object is independent of the path taken.

In both the paths the work done by the gravitational force is 10,000 J

Along path (1): more force needs to be applied against gravity to cover lesser distance .

Along path (2): lesser force needs to be applied against the gravity to cover more distance.

As the force needs to be applied along the ramp is less, it is easier to move the object along the ramp.

Example 4.15

An object of mass m is projected from the ground with initial speed v_0.

Find the speed at height h.

Solution

Since the gravitational force is conservative; the total energy is conserved throughout the motion.

	Initial	Final
Kinetic energy	$\frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2$
Potential energy	0	mgh
Total energy	$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_0^2$	$\frac{1}{2}mv^2 + mgh$

Final values of potential energy, kinetic energy and total energy are measured at the height *h*.

By law of conservation of energy, the initial and final total energies are the same.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh$$

$$v_0^2 = v^2 + 2gh$$

$$v = \sqrt{v_0^2 - 2gh}$$

Note that in section (2.11.2) similar result is obtained using kinematic equation based on calculus method. However, calculation through energy conservation method is much easier than calculus method.

Solved Example Problems for Unit of power

Example 4.18

Calculate the energy consumed in electrical units when a 75 W fan is used for 8 hours daily for one month (30 days).

Solution

Power, P = 75 W

Time of usage, t = 8 hour × 30 days = 240 hours

Electrical energy consumed is the product of power and time of usage.

Electrical energy = power × time of usage = P × t

$$= 75 \text{ watt} \times 240 \text{ hour}$$

$$= 18000 \text{ watt hour}$$

$$= 18 \text{ kilowatt hour} = 18 \text{ kWh}$$

$$1 \text{ electrical unit} = 1 \text{ kWh}$$

$$\text{Electrical energy} = 18 \text{ unit}$$

Solved Example Problems for Relation between power and velocity

Example 4.19

A vehicle of mass 1250 kg is driven with an acceleration 0.2 ms^{-2} along a straight level road against an external resistive force 500 N.Calculate the power delivered by the vehicle's engine if the velocity of the vehicle is 30 m s^{-1} .

Solution

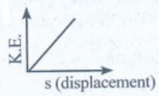
The vehicle's engine has to do work against resistive force and make vechile to move with an acceleration. Therefore, power delivered by the vehicle engine is

P=F*V=500*30W=15kW

4. A car starts from rest and moves on a surface with uniform acceleration.

Draw the graph of kinetic energy versus displacement. What information you can get from that graph?

Answer



$$\text{K.E.} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\,(2\,as) \quad \text{[From equation of motion}$$

$\therefore$ K.E. $\propto$ S.