

form is called as a MAX TERM.

\* A max term gives the value '0' for exactly one combination of variables.

\* The product of all max term of 'f' for which 'f' assumes '0' is called canonical product of sums or conjunctive normal forms.

lecture 02

### Redundant literal Rule

$$1) A + \bar{A}B = A + B$$

$$2) A(\bar{A} + B) = AB \quad \left. \begin{array}{l} 1) \\ 2) \end{array} \right\} \text{dual form}$$

### Absorption Law

$$1) A + A \cdot B = A$$

$$2) A(A+B) = A$$

### Consensus Theorem

$$1) AB + \bar{A}C + BC = AB + \bar{A}C$$

$$2) (A+B)(\bar{A}+C)(B+C) \\ = (A+B)(\bar{A}+C)$$

### Transposition Theorem

$$AB + \bar{A}C = (A+C)(\bar{A}+B)$$

### Consensus Theorem

$$\begin{aligned} \textcircled{1} \text{ L.H.S } & AB + \bar{A}C + BC \\ &= AB + \bar{A}C + BC(A + \bar{A}) \\ &= AB + \bar{A}C + ABC + \bar{A}BC \\ &= AB(1+C) + \bar{A}C(1+B) \\ &= AB + \bar{A}C = \text{R.H.S} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \text{ L.H.S } & (A+B)(\bar{A}+C)(B+C) \\ &= (A\bar{A} + AC + B\bar{A} + BC)(B+C) \\ &= (0 + AC + B\bar{A} + BC)(B+C) \\ &= ABC + B\bar{A}B + B\bar{A}C + ACB \\ &\quad + ACC + B\bar{A}C + BCC \\ &= ABC + \bar{A}B + BC + AC + \bar{A}BC \\ &\quad + BC \end{aligned}$$

$$= BC(A+B) + \bar{A}B(1+C) + BC + AC$$

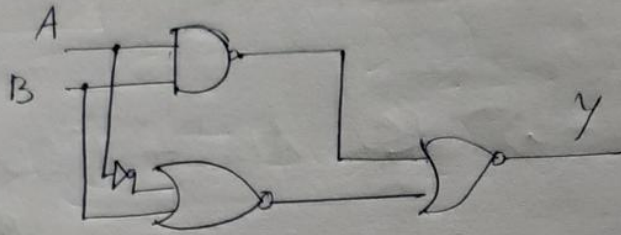
$$= BC + BC + \bar{A}B + AC$$



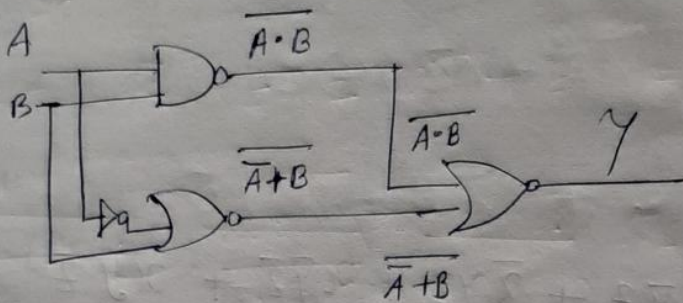
# Boolean Algebra & Expression - Part IX

[NET-DEC-2018]

Q

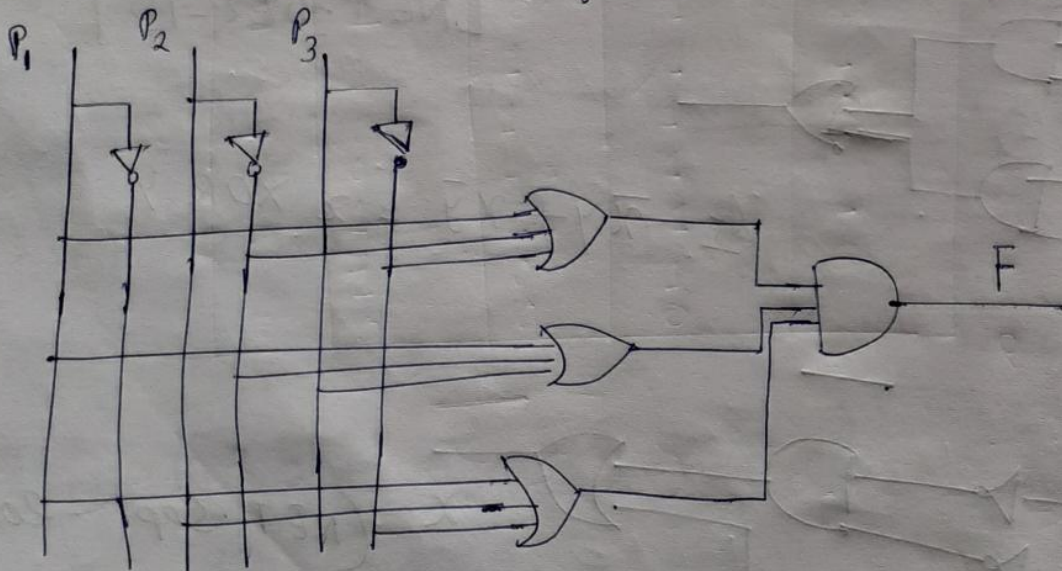


Soln: -



$$= \overline{A \cdot B} + \overline{A + B} = \overline{A \cdot B} \cdot \overline{A + B} = A \cdot B (\bar{A} + \bar{B}) = AB \neq$$

Q What is the O/P? [NET-NOV-2017]



$$\text{Soln } (P_1 + \bar{P}_2 + \bar{P}_3) (P_1 + \bar{P}_2 + P_3) (P_1 + P_2 + \bar{P}_3)$$

$$= \sum m(3, 2, 1) \approx \sum m(0, 4, 5, 6, 7)$$

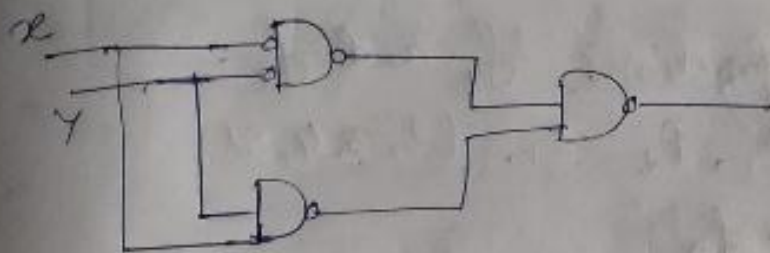
$$= \bar{P}_1 \bar{P}_2 \bar{P}_3 + P_1 \bar{P}_2 \bar{P}_3 + P_1 \bar{P}_2 P_3 + \bar{P}_1 P_2 \bar{P}_3$$

$$+ P_1 P_2 P_3$$

$$\begin{aligned}
 &= P_1 \bar{P}_2 (P_3 + \bar{P}_3) + P_1 P_2 (\bar{P}_3 + P_3) + \bar{P}_1 P_2 \bar{P}_3 \\
 &= P_1 \bar{P}_2 + P_1 P_2 + \bar{P}_1 \bar{P}_2 P_3 = \bar{P}_2 (P_1 + \bar{P}_1 P_3) + P_1 P_2 \\
 &= \bar{P}_2 (P_1 + \bar{P}_3) + P_1 P_2 = P_1 \bar{P}_2 + \bar{P}_2 \bar{P}_3 + P_1 P_2 = P_1 (P_2 + \bar{P}_3) + \bar{P}_2 \bar{P}_3 \\
 &= P_1 + \bar{P}_2 \bar{P}_3 \quad \#
 \end{aligned}$$

Q what is the o/p of the following circuit-

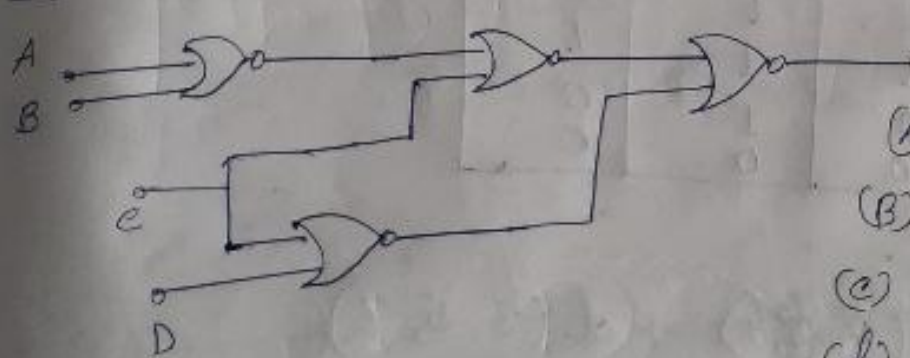
[NET-AUG-2010]



Soln -  $\overline{\overline{X \cdot Y} \cdot \overline{X \cdot Y}} = \overline{\overline{X \cdot Y}} + \overline{\overline{X \cdot Y}}$

$$= X \cdot Y + X \cdot Y = X \odot Y \quad \#$$

Q what is the o/p? [NET - June - 2010]



- options
- (A)  $\bar{A}\bar{C} + \bar{B}\bar{C} + CD$
  - (B)  $A\bar{C} + B\bar{C} + \bar{C}D$
  - (C)  $ABC + \bar{C}D$
  - (D)  $\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}D$

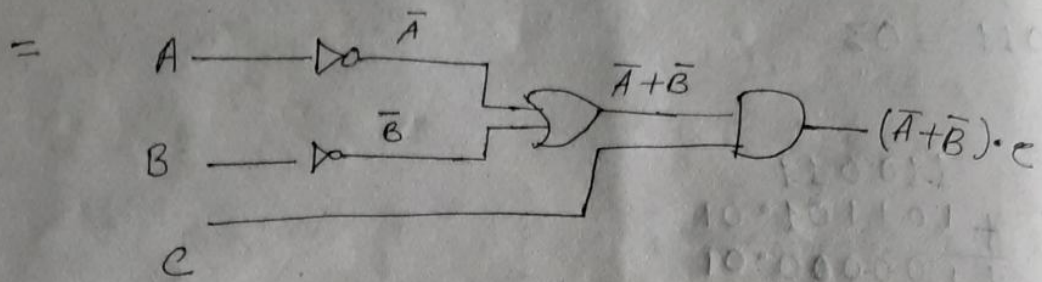
Soln

$$\begin{aligned}
 &= \overline{\overline{a \cdot b + c} + (c+d)} \\
 &= \overline{\overline{a \cdot b + c}} \cdot \overline{c+d} = (a \cdot b + c)(c+d) \\
 &= \bar{a}\bar{b}c + \bar{a}\bar{b}d + c + cd = \bar{a}\bar{b}d + c
 \end{aligned}$$

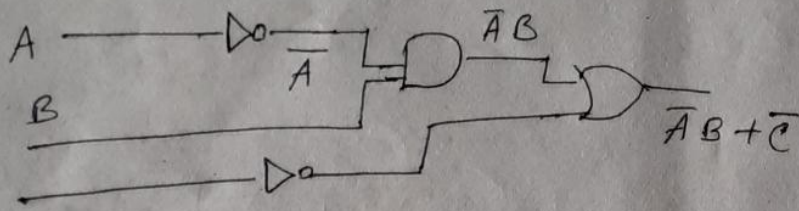
But it is not matching with any of the options even after drawing K-Map.



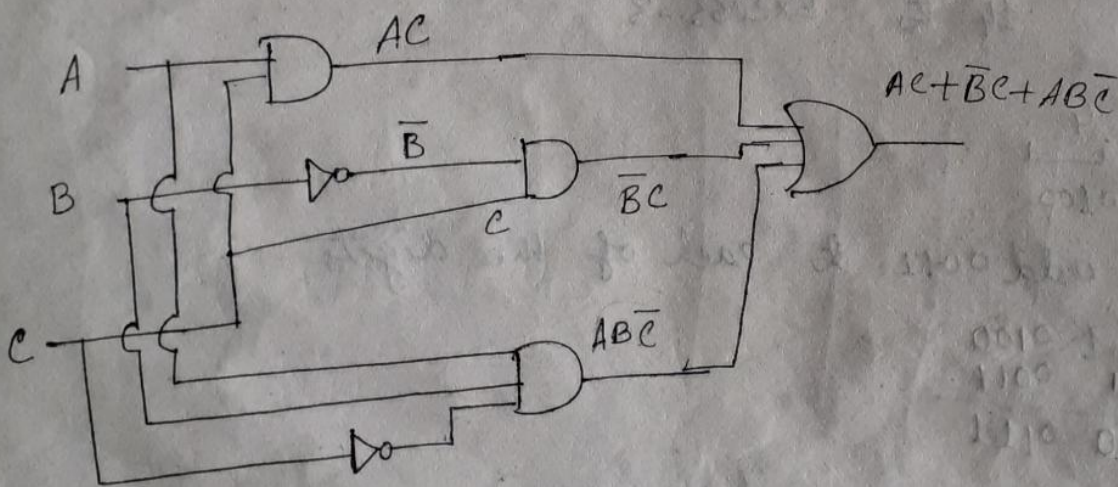
Q  $y = (\bar{A} + \bar{B}) \cdot c$



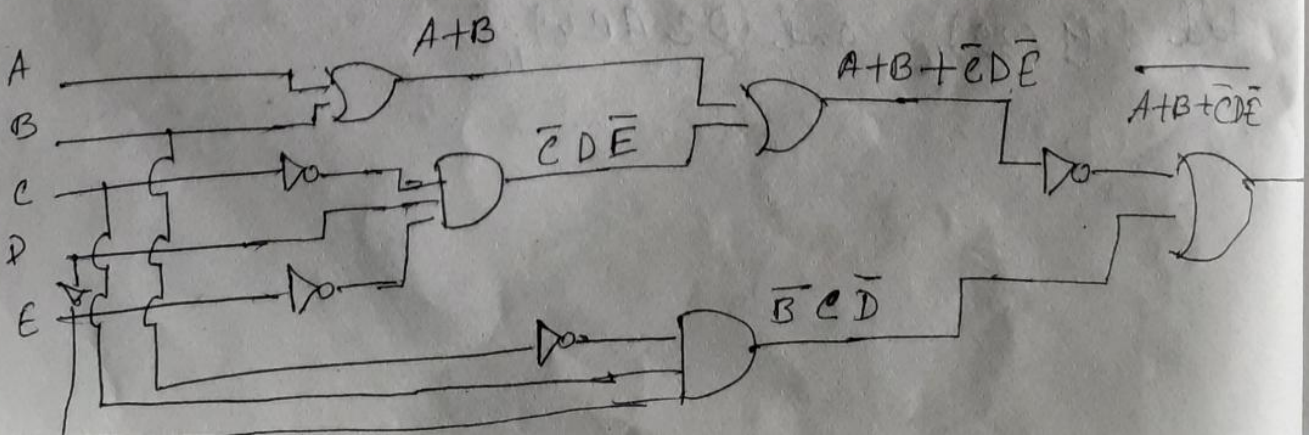
Q  $y = \bar{A}B + \bar{c}$



Q  $y = AC + \bar{B}C + AB\bar{c}$



Q  $y = \overline{(A+B+\bar{c}D\bar{E})} + \bar{B}C\bar{D}$



## Boolean Algebra

### ① Idempotent Law

$$a \cdot a = a$$

$$a + a = a$$

### ③ Commutative Law

$$a \cdot b = b \cdot a$$

$$a + b = b + a$$

### ⑤ De Morgan's Law

$$\overline{(a + b)} = \bar{a} \cdot \bar{b}$$

$$\overline{a \cdot b} = \bar{a} + \bar{b}$$

### ⑦ Complement Law

$$① \bar{\bar{a}} = a$$

$$③ a \cdot \bar{a} = 0$$

$$② \bar{\bar{\bar{a}}} = \bar{a}$$

$$④ a + \bar{a} = 1$$

### ② Associative Law

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

$$a + (b + c) = (a + b) + c$$

### ④ Distributive Law

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

### ⑥ Identity Law

$$a + 0 = a$$

$$a \cdot 0 = 0$$

$$a + 1 = 1$$

$$a \cdot 1 = a$$

### ⑧ Involution Law

$$\overline{(\bar{a})} = a$$



# Proof of De-Morgan Theorem

$$\textcircled{1} \quad \overline{A \cdot B} = \overline{A} + \overline{B}$$

From Complementary Law

$$A + \overline{A} = 1, \quad A \cdot \overline{A} = 0 \quad \phi$$

$$(x+y) + (\overline{x+y}) = 1$$

$$(x+y) + \overline{x} \cdot \overline{y}$$

$$(x+y+\overline{x})(x+y+\overline{y})$$

$$= (1+y)(1+x)$$

$$= 1 \cdot 1 = 1$$

$$(x+y) \cdot (\overline{x+y}) = 0$$

$$(x+y) \cdot (\overline{x} \cdot \overline{y})$$

$$(x \cdot \overline{x} \cdot \overline{y}) + (y \cdot \overline{x} \cdot \overline{y})$$

$$= (0 \cdot \overline{y}) + (0 \cdot \overline{x})$$

$$= 0 + 0$$

$$= 0$$

$$\textcircled{2} \quad \overline{\overline{x} \cdot \overline{y}} + x \cdot y + \overline{x} \cdot y \quad [\text{Simplify the expression}]$$

$$= \overline{\overline{x} \cdot \overline{y}} + y(x + \overline{x})$$

$$= \overline{\overline{x} \cdot \overline{y}} + y \cdot 1 \quad [\text{Complement Law}]$$

$$= \overline{\overline{x} \cdot \overline{y}} + y \quad [\text{Identity Law}]$$

$$= (\overline{\overline{x}} + y)(\overline{\overline{y}} + y) \quad [\text{Distributive Law}]$$

$$= (\overline{\overline{x}} + y) \cdot 1 \quad [\text{Complement Law}]$$

$$= (\overline{\overline{x}} + y) \quad [\text{Identity Law}]$$