classmate

$$= b\int_0^h \left(\frac{h-y}{h}\right) y^2\, dy$$

$$= b\int_0^h \left(y^2 - \frac{y^3}{h}\right) dy$$

$$= b\left[\int_0^h y^2\, dy - \int_0^h \frac{y^3}{h}\, dy\right]$$

$$= b\left[\left(\frac{y^3}{3}\right)_0^h - \frac{1}{h}\left(\frac{y^4}{4}\right)_0^h\right]$$

$$= b\left[\frac{h^3}{3} - \frac{1}{h}\frac{h^4}{4}\right]$$

$$= b\left[\frac{h^3}{3} - \frac{h^3}{4}\right]$$

$$\cancel{= b\,[}$$

$$= b \times \frac{h^3}{12}$$

$$\therefore \boxed{I_{AB} = \frac{bh^3}{12}}$$

$$\therefore I_G = I_{AB} - ad^2$$

$$= \frac{bh^3}{12} - \left(\frac{bh}{2}\right) \times \left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12} - \frac{bh}{2} \times \frac{h^2}{9}$$

$$= \frac{bh^3}{12} - \frac{bh^3}{18}$$

classmate

$$I_G = bh^3\left(\frac{1}{12} - \frac{1}{18}\right)$$

$2 \,|\, 12, 18$
$3 \,|\, 6, 9$
$2, 3$

$$= bh^3\left(\frac{3-2}{36}\right)$$

$$= bh^3 \times \frac{1}{36}$$

$$\therefore \boxed{I_G = \frac{bh^3}{36}}$$

$$I_{ST} = I_G + ay^2$$

$$= \frac{bh^3}{36} + \left(\frac{bh}{2}\right) \times \left(\frac{2h}{3}\right)^2$$

$$= \frac{bh^3}{36} + \frac{bh}{2} \times \frac{4h^2}{9}$$

$$= \frac{bh^3}{36} + \frac{4bh^3}{18}$$

$$= bh^3\left(\frac{1 + 4\times 2}{36}\right)$$

$$= \frac{bh^3}{\cancel{60}}\left(\frac{9}{36}\right)$$

$$= \frac{9bh^3}{36}$$

$$\therefore \boxed{I_{ST} = \frac{bh^3}{4}}$$

Q. An issosceles trianglular section ABC has base with 80mm and height 60mm. Determine the MI of the section about CG and base.

Sol^n Given, $b = 80$ mm
$h = 60$ mm

$$I_G = \frac{bh^3}{36}$$

$$= \frac{80 \times 60^3}{36}$$

$$= 480000$$

$$= 48 \times 10^4 \ m^4$$

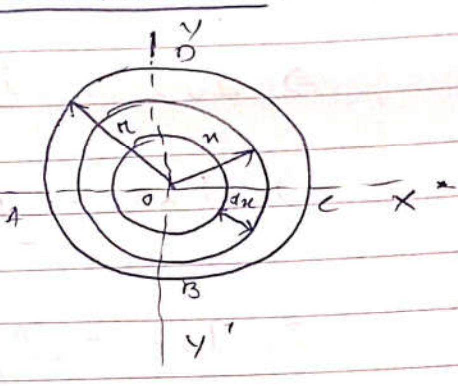
$$I_{AB} = \frac{bh^3}{12}$$

$$= \frac{80 \times 60^3}{12}$$

$$= 1440000$$

$$= 144 \times 10^4 \ m^4$$

Moment Of Inertia Of a Circular Section



Consider a circle ABCD of radius r with centre O and XX' and YY' be the two axis of reference through O as shown in figure.

Now, consider an elemental ~~strip~~ ring of radius x and thickness dx. Therefore area of the ring dA = $2\pi x \times dx$

Moment of inertia of the ring about ZZ axis

$$= (2\pi x \times dx) \times x^2$$

$$= 2\pi x^3 \, dx$$

$$\therefore I_{ZZ} = \int_0^r 2\pi x^3 \, dx$$

$$= 2\pi \left[\frac{x^4}{4}\right]_0^r$$

$$= \frac{2\pi}{4} r^4$$

$$= \frac{\pi r^4}{2}$$

$$= \frac{\pi d^4}{32}$$

From the perpendicular axis theorem

$$I_{ZZ} = I_{XX} + I_{YY}$$

From the figure $I_{XX} = I_{YY}$

$\therefore I_{zz} = 2I_{xx}$

$\Rightarrow I_{xx} = \frac{I_{zz}}{2}$

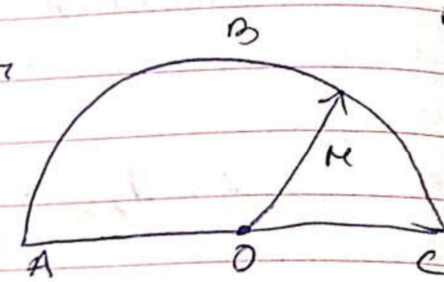
$\Rightarrow I_{xx} = \frac{\frac{\pi d^4}{32}}{2}$

$\therefore I_{xx} = \frac{\pi d^4}{64} = I_{yy}$

<u>Moment of Inertia of a Semi-Circular Section</u>

~~Consider~~

Let us consider a semi-circular section ABC whose moment of inertia is required to be found out as shown in figure.



Let 'r' = radius of the semi-circle

Moment of inertia of the semi-circular section about the base AC

$I_{AC} = \frac{1}{2} \frac{\pi d^4}{64}$

$$\boxed{I_{AC} = \frac{\pi d^4}{128} = 0.393\, r^4}$$

$I_G = I_{AC} - ad^2$

$= \frac{\pi d^4}{128} - \frac{\pi d^2}{8} \times d^2$

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$$I_G = \frac{\pi d^4}{128} - \frac{\pi d^4}{8} \times \left(\frac{4m}{3\pi}\right)^2$$

$0.11m^4$