

Co-ordinate Geometry in 2-D

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Equations of Circle :-

(i) When centre $C(h, k)$ and radius ' r ' is given, then, equation of circle is —

$$(x-h)^2 + (y-k)^2 = r^2 \longrightarrow (1)$$

⇒ When centre is at origin,

$C(0,0)$

$$(x-0)^2 + (y-0)^2 = r^2$$

$$\Rightarrow x^2 + y^2 = r^2$$

⇒ When centre is at origin $C(0,0)$ and radius = 1 unit

$$(x-0)^2 + (y-0)^2 = 1^2$$

$$\Rightarrow x^2 + y^2 = 1 \Rightarrow \text{It is known as unit circle.$$

⇒ Standard form or General equation of a circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0 \longrightarrow (2)$$

Here, Centre = $(-g, -f)$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

* Equation in $x^2 + y^2 + 2gx + 2fy + c = 0$ circle if eqn is (must be)

⇒ When the centre at origin

⇒ If circle passes through the origin,

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$x^2 + y^2 + 2gx + 2fy = 0$$

ii) If centre lies on x-axis, then,

$$x^2 + y^2 + 2gx + c = 0$$

When centre lies on y-axis, then,

$$x^2 + y^2 + 2fy + c = 0$$

iii) If $g^2 + f^2 > c$ (is greater than) c , then, eqn (2), represents a circle with centre $C(-g, -f)$ and radius is real. ~~$x^2 + y^2$~~

iv) If $g^2 + f^2$ is less than c ($<$) than eqn (2) represents a circle with centre $C(-g, -f)$ and radius is imaginary and it is called virtual circle.

v) If $g^2 + f^2$ is equal to c , then eqn (2) represents a circle with centre $C(-g, -f)$ and radius is equal to zero (0) i.e. co-inside with the circle and it is called point circle.

Q) Find the eqn. of the circle with the centre $C(-2, 1)$ and radius 3.

Soln: Given, $C(-2, 1)$

$$r = 3.$$

We know that,

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\Rightarrow (x+2)^2 + (y-1)^2 = 3^2$$

$$\Rightarrow x^2 + 2 \cdot x \cdot 2 + 2^2 + y^2 - 2 \cdot 1 \cdot y + 1 = 9$$

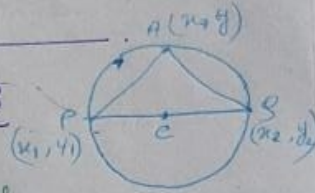
$$\Rightarrow x^2 + 4x + 4 + y^2 - 2y + 1 = 9$$

$$\Rightarrow x^2 + y^2 + 4x - 2y + 5 = 9$$

$$\Rightarrow x^2 + y^2 + 4x - 2y = 9 - 5$$

$$\Rightarrow x^2 + y^2 + 4x - 2y = 4$$

(3) Circle with given diameter :



When 2 ends points $P(x_1, y_1)$ and $Q(x_2, y_2)$ of diameter is given, then eqn is -

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0 \quad \text{****}$$

(3) Find the equation of the circle of which the points $(5, -3)$ and $(-1, -6)$ are the end points of a diameter.

Soln:- $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$$\Rightarrow (x - 5)(x - (-1)) + (y - (-3))(y - (-6)) = 0$$

$$\Rightarrow x^2 - x - 5x + 5 + y^2 + 6y + 3y + 18 = 0$$

$$\Rightarrow x^2 + y^2 - 6x + 5 + 9y + 18 = 0$$

$$\Rightarrow x^2 + y^2 + 9y - 6x + 23 = 0 \quad \text{✓}$$