which is the required equation of circle.

SOLVED EXAMPLES

Example 12.1. Find the equation of a circle whose circle is at (0, 2) and radius = 2.

Solution. Let the required equation of circle be

$$(x-h)^2+(y-k)^2 = r^2 \quad \text{...(i)}$$

Given, centre $= (h, k) = (0, 2)$

r = radius = 2

∴ From Eq. (*i*)

$$(x-0)^2+(y-2)^2 = (2)^2$$

$$\Rightarrow \quad x^2+y^2+4-4y = 4$$

$$\Rightarrow \quad x^2+y^2-4y = 0$$

be the required equation.

Example 12.2. Find centre and radius of following circles

(*i*) $x^2+y^2-12x+6y+45=0$ (*ii*) $(x+5)^2+(y-3)^2=36$

(*iii*) $2x^2+2y^2-6x+8y-4=0$ (*iv*) $4(x^2+y^2)-10x+5y+\frac{5}{4}=0.$

Solution. (*i*) The equation of a given circle is

$$x^2+y^2-12x+6y+45 = 0$$

Comparing with $x^2+y^2+2gx+2fy+c=0$

$$\therefore \quad 2g = -12 \Rightarrow g = -6$$

$$2f = 6 \Rightarrow f = 3$$

$$c = 45$$

∴ Co-ordinates of centre are $= (-g, -f) = (6, -3)$

and Radius of circle $= \sqrt{g^2+f^2-c} = \sqrt{(6)^2+(-3)^2-45} = \sqrt{36+9-45} = 0$ units

(*ii*) The equation of given circle is

$$(x+5)^2+(y-3)^2 = 36 = (6)^2$$

Comparing it with

$$(x-h)^2+(y-k)^2 = r^2$$

∴ Centre of circle is $= (h, k) = (-5, 3)$

∴ Radius = 6 units

(*iii*) The equation of given circle is

$$2x^2+2y^2-6x+8y-4 = 0 \quad \text{...(i)}$$

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[∵ As we know, to make the equation of circle, coefficient of x^2 and y^2 must be unity]

∴ Dividing Eq. (*i*) by 2

$$x^2 + y^2 - 3x + 4y - 2 = 0$$

∴ Now comparing it with equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

∴ $2g = -3 \Rightarrow g = \frac{-3}{2}$ and $2f = 4 \Rightarrow f = 2,\ c = -2$

∴ Coordinates of centre $= (-g, -f) = \left[-\left(\frac{-3}{2}\right), -2\right] = \left(\frac{3}{2}, -2\right)$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{3}{2}\right)^2 + (-2)^2 - (-2)}$$

$$= \sqrt{\frac{9}{4} + \frac{4}{1} + 2} = \sqrt{\frac{9+16+8}{4}} = \frac{\sqrt{33}}{2} \text{ units}$$

(*iv*) The equation of given circle is

$$4(x^2 + y^2) - 10x + 5y + \frac{5}{4} = 0 \quad ...(i)$$

Dividing Eq. (*i*) by 4

$$x^2 + y^2 - \frac{10}{4}x + \frac{5}{4}y + \frac{5}{4} = 0$$

Comparing it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

∴ $2g = \frac{-10}{4} \Rightarrow g = \frac{-10}{4 \times 2} = \frac{-5}{4}$

∴ $2f = \frac{5}{4} \Rightarrow f = \frac{5}{8}$

$c = \frac{5}{4}$

∴ Centre $= (-g, -f) = \left(\frac{5}{4}, \frac{-5}{8}\right)$

$$\text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{5}{4}\right)^2 + \left(\frac{-5}{8}\right)^2 - \frac{5}{4}}$$

$$= \sqrt{\frac{25}{16} + \frac{25}{64} - \frac{5}{4}} = \sqrt{\frac{45}{64}} = \frac{3\sqrt{5}}{8} \text{ unit.}$$

Example 12.3. Find the equation of the circle whose centre is (2, –1) and passes through (3, 6).

Solution. Given $C(h, k) = (2, -1)$

and $P(x, y) = (3, 6)$

∴ Radius $= |CP|$

$$= \sqrt{(3-2)^2 + (6+1)^2}$$

$$= \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

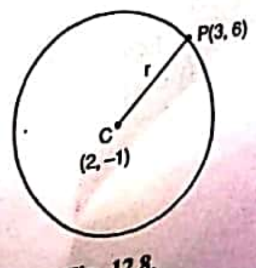

Fig. 12.8.

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Let the equation of circle be

$$(x-h)^2+(y-k)^2 = r^2 \quad ...(i)$$

Now put $C(h, k) = C(2, -1)$ and $r = 5\sqrt{2}$ in Eq. (i)

$$(x-2)^2+(y+1)^2 = (5\sqrt{2})^2$$

$\Rightarrow \quad x^2+4-4x+y^2+1+2y = 50$

$\Rightarrow \quad x^2+y^2-4x+2y-45 = 0$ be required equation of circle.

Example 12.4. Find the equation of the circle of radius 5 whose centre lies on y-axis and passes through (3, 2).

Solution. Since centre of required circle lies on y-axis. So, its coordinates are $(0, k)$

Then equation of circle is

$$(x-0)^2+(y-k)^2 = (5)^2$$

$\Rightarrow \quad x^2+y^2+k^2-2ky-25 = 0 \quad ...(i)$

Equation (i) passes through $P(3, 2)$

So $(3)^2+(2)^2+k^2-2k(2)-25 = 0$

$\Rightarrow \quad 9+4-4k+k^2-25 = 0$

$\Rightarrow \quad k^2-4k-12 = 0$

$k^2-6k+2k-12 = 0$

$k(k-6)+2(k-6) = 0$

$(k-6)(k+2) = 0$

$k = 6, -2$

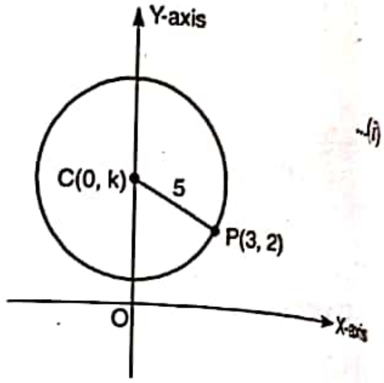

Fig. 12.9.

(i) When $k = 6$, then Eq. (i) becomes

$$(x-0)^2+(y-6)^2 = 25$$

$x^2+y^2+36-12y-25 = 0$

$\Rightarrow \quad x^2+y^2-12y+11 = 0$

(ii) When $k = -2$, Eq. (i) becomes

$$(x-0)^2+(y+2)^2 = (5)^2$$

$x^2+y^2+4+4y = 25$

$\Rightarrow \quad x^2+y^2+4y-21 = 0$

Example 12.5. Find the equation of a circle whose centre is (4, 5) and which passes through centre of the circle, $x^2+y^2+4x-6y = 12$.

Solution. Equation of given circle is

$$x^2+y^2+4x-6y-12 = 0$$

Comparing it with

$$x^2+y^2+2gx+2fy+c = 0$$

$\therefore$ Centre $= (-g, -f) = (-2, 3)$

$\therefore$ Equation of required circle whose centre is (4, 5) and passes through (–2, 3) is

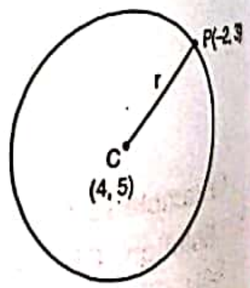

Fig. 12.10.

$\because$

$$r = |CP|$$

$$= \sqrt{(4+2)^2+(5-3)^2} = \sqrt{36+4} = \sqrt{40}$$

$$= 2\sqrt{10} \text{ units}$$

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$$(x-4)^2 + (y-5)^2 = (2\sqrt{10})^2$$

$\Rightarrow \quad x^2 + 16 - 8x + y^2 + 25 - 10y =$

$\Rightarrow \quad x^2 + y^2 - 8x - 10y + 1 = 0$ be required equation of circle. 40

Example 12.6. Find the equation of a circle passing through the points (5, 7), (6, 6) and (2, –2). Also find its centre and radius.

Solution. Let equation of required circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad ...(i)$$

Since 1 passes through (5, 7). So, it satisfy Eq. (*i*)

$\therefore \quad 25 + 49 + 10g + 14f + c = 0$

$\Rightarrow \quad 74 + 10g + 14f + c = 0 \quad ...(ii)$

Equation (*i*) passes through (6, 6)

$\therefore \quad 36 + 36 + 12g + 12f + c = 0$

$\Rightarrow \quad 72 + 12g + 12f + c = 0 \quad ...(iii)$

Also Eq. (*i*) passes through (2, –2)

$\therefore \quad 4 + 4 + 4g - 4f + c = 0$

$\Rightarrow \quad 8 + 4g - 4f + c = 0 \quad ...(iv)$

$(iii) - (ii) \Rightarrow \quad -2 + 2g - 2f = 0$

$\Rightarrow \quad g - f = 1 \quad ...(v)$

$(iv) - (iii) \Rightarrow \quad -64 - 8g - 16f = 0$

$\Rightarrow \quad g + 2f + 8 = 0 \quad ...(vi)$

$(v) - (vi) \Rightarrow \quad -3f - 9 = 0 \Rightarrow f = 3$

Put the value of $f = -3$ in Eq. (*v*)

$$g + 3 = 1 \Rightarrow g = -2$$

Putting $g = -2, f = -3$ in Eq. (*iv*)

$\Rightarrow \quad 8 - 8 + 12 + c = 0$

$\Rightarrow \quad c = -12$

Putting value of g, f, c in Eq. (*i*)

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Comparing it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$\therefore \quad 2g = -4 \Rightarrow g = -2$

$2f = -6 \Rightarrow f = -3 \quad \therefore \quad c = -12$

Centre $= (-g, -f) = (2, 3)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$ units

Example 12.7. Find equation of circle concentric (same centre) with the circle $x^2 + y^2 - 4x - 6y - 9 = 0$ and passes through point $P(-4, -5)$.

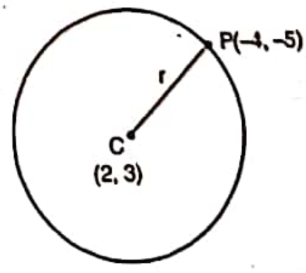

Fig. 12.11.

Solution. Equation of given circle is

$$x^2 + y^2 - 4x - 6y - 9 = 0$$

Compare it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$\therefore \quad 2g = -4 \Rightarrow g = -2$

$2f = -6 \Rightarrow f = -3$

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$$c = -9$$

$$\text{Centre} = (-g, -f) = (2, 3)$$

$$\therefore \quad r = |CP| = \sqrt{(2+4)^2 + (3+5)^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$

$\therefore$ Equation of required circle having centre (2, 3) and radius 10 is

$$(x-2)^2 + (y-3)^2 = (10)^2$$

$$x^2 + 4 - 4x + y^2 + 9 - 6y = 100$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 87 = 0$$

be required equation of circle.

Example 12.8. Find the equation of circle which passes through the points (3, 7), (5, 5) and whose centre lies on the line $x - 4y - 1 = 0$.

Solution. Let the equation of required circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{...(i)}$$

$\because$ Eq. (*i*) passes through (5, 5), so it satisfy the Eq. (*i*)

$$\therefore \quad 25 + 25 + 10g + 10f + c = 0$$

$$\Rightarrow \quad 50 + 10g + 10f + c = 0 \quad \text{...(ii)}$$

$\therefore$ Eq. (*ii*) passes through (3, 7)

$$\therefore \quad 9 + 49 + 6g + 14f + c = 0$$

$$\Rightarrow \quad 58 + 6g + 14f + c = 0 \quad \text{...(iii)}$$

Note: Now centre $(-g, -f)$ lies on line $x - 4y - 1 = 0$

$$\therefore \quad -g + 4f - 1 = 0 \quad \text{...(iv)}$$

$$(ii) - (iii) \Rightarrow \quad 4g - 4f - 8 = 0$$

$$g - f - 2 = 0 \quad \text{...(v)}$$

$$(iv) + (v) \Rightarrow \quad 3f - 3 = 0 \Rightarrow f = 1$$

Put $f = 1$ in Eq. (*v*)

$$g - 1 - 2 = 0$$

$$g = 3$$

Put $f = 1$, $g = 3$ in Eq. (*ii*)

$$50 + 10(3) + 10(1) + c = 0$$

$$\Rightarrow \quad c = -90$$

Putting value of g, f and c in Eq. (*i*)

$$x^2 + y^2 + 6x + 2y - 90 = 0$$

is required equation of circle.

Example 12.9. Prove that points (1, –6), (5, 2), (7, 0) and (–1, –4) are concyclic. Find radius of circle.

Solution. Let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{...(i)}$$

Since Eq. (*i*) passes through point (1, –6)

$$\therefore \quad 1 + 36 + 2g - 12f + c = 0$$

$$\Rightarrow \quad 37 + 2g - 12f + c = 0 \quad \text{...(ii)}$$

Since Eq. (*i*) passes through (5, 2)

$$\therefore \quad 25 + 4 + 10g + 4f + c = 0$$

$$\Rightarrow \quad 29 + 10g + 4f + c = 0 \quad \text{...(iii)}$$

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Since Eq. (*i*) passes through (7, 0)

$$49 + 0 + 14g + 0 + c = 0$$

$$\therefore \quad 49 + 14g + c = 0 \qquad \text{...(iv)}$$

Eqs. (*iii*) – (*ii*) $\Rightarrow -8 + 8g + 16f = 0$

$$-1 + g + 2f = 0 \qquad \text{...(v)}$$

(*iv*) – (*iii*) $\Rightarrow$ $20 + 4g - 4f = 0$

$$5 + g - f = 0 \qquad \text{...(vi)}$$

(*vi*) – (*v*) $\Rightarrow$ $6 - 3f = 0$

$$3f = 6 \Rightarrow f = 2$$

Put $f = 2$ in Eq. (*vi*)

$$5 + g - 2 = 0$$

$$g = -3$$

Put value of g, f in Eq. (*iii*)

$$29 + 10(-3) + 4(2) + c = 0$$

$$\Rightarrow \quad 29 - 30 + 8 + c = 0$$

$$\Rightarrow \quad c = -7$$

Put value of g, f, c in Eq. (*i*)

$$x^2 + y^2 + 2(-3)x + 2(2)y - 7 = 0$$

$$x^2 + y^2 - 6x + 4y - 7 = 0 \qquad \text{...(vii)}$$

Putting the coordinates of fourth point (–1, –4) in Eq. (*vii*)

$$\Rightarrow \quad (-1)^2 + (-4)^2 - 6(-1) + 4(-4) - 7 = 0$$

$$\Rightarrow \quad 1 + 16 + 6 - 16 - 7 = 0$$

$\Rightarrow \quad 0 = 0$, which is true.

Hence given four points are concyclic.

$$\therefore \quad \text{Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{9+4+7} = \sqrt{20} = 2\sqrt{5} \text{ unit}$$

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